NetSurvival.jl: A glimpse into relative survival analysis.

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[Introduction to relative survival](#page-2-0) [analysis](#page-2-0)

Survival Analysis is a statistical theory that deals with censored positive random variables:

- (i) Events (usually death) can occur or not before censoring (exit of experiment)
- (ii) We want to model the time to event (survival)
- (iii) Relies on counting processes and local martingales.

Relative survival stochastic settings

Relative Survival Context: In population-based studies, the specific cause of death is often unidentified, unreliable or even unavailable. We then work with the following random variables:

Notations: We use both $S_X(t) = \mathbb{P}(X > t)$, $\Lambda_X = -\ln S_X$ or $\lambda_X = \partial \Lambda_X$ to characterize a distribution.

Goal: Estimate the distribution of E, say by it's hazard $\partial \Lambda_E(t) = -\partial \ln S_E(t)$.

Remark: Without the cause of death indicatrix, standard competing risks models cannot be used...

Remark: The joint distribution of (E, P, C, D) characterizes our observations.

Assumptions (Standard independence $^1)$

- (i) C, E and (P, D) are mutually independent
- (ii) The distribution of $P|\mathbf{D}$ is known from standard life tables (at time 0) for each individual.

(iii) Individuals are i.i.d.

Note: The population hazard for each individual λ_{P_i} is drawn from a reference <code>RateTable</code> from RateTables. jl and may depend on covariates $D_1, ..., D_n$ such as age, year, sex, country, race, etc...

¹Maja Pohar Perme, Janez Stare, and Jacques Estève. "On Estimation in Relative Survival". In: Biometrics 68.1 (June 2011), pp. 113–120. ISSN: 0006-341X. DOI: [10.1111/j.1541-0420.2011.01640.x](https://doi.org/10.1111/j.1541-0420.2011.01640.x). [Standard estimators in the field](#page-6-0)

The estimation of net survival is usually discussed in terms of the estimation of the cumulative excess hazard $\Lambda_F(t)$ and/or the instantaneous hazard $\lambda_F = \partial \Lambda_F$.

Definition (Pohar Perme 2012²)

The **Pohar Perme** estimator of the excess hazard is given, using $w_i(s) = S_{P_i}(s)^{-1}$, by:

$$
\partial \hat{\Lambda}_E(s) = \frac{\sum_{i=1}^n w_i(s) \left\{ \partial N_i(s) - Y_i(s) \partial \Lambda_{P_i}(s) \right\}}{\sum_{i=1}^n w_i(s) Y_i(s)}
$$

and its variance can be estimated by:

$$
\partial \hat{\sigma}_E^2(s) = \frac{\sum_{i=1}^n w_i(s)^2 \partial N_i(s)}{\left(\sum_{i=1}^n w_i(s)^2 Y_i(s)\right)^2}
$$

Good properties: This estimator is biased but convergent and asymptotically unbiased.

²Maja Pohar Perme, Janez Stare, and Jacques Estève. "On Estimation in Relative Survival". In: Biometrics 68.1 (June 2011), pp. 113–120. ISSN: 0006-341X. DOI: [10.1111/j.1541-0420.2011.01640.x](https://doi.org/10.1111/j.1541-0420.2011.01640.x).

Sidenote: Unicode is beautiful.

One big advantage with Julia is the how easy to read and comprehensible the syntax is.

```
const PoharPerme = NPNSEstimator{PoharPermeMethod}
```

```
function Λ!(::Type{PoharPermeMethod}, ∂N., Y., ∂N., Y., ∂V. T. Δ. age, date, rate preds, ratetable, grid, ∂t)
for i in eachindex(age)
     T_i = searchsortedlast(grid, T[i])
     \Lambda_n, W_n, rt; = 0.0, 1.0, ratetable[rate preds[i.:]...]
     for j in 1:T_i\lambda<sub>p</sub> = daily hazard(rt<sub>i</sub>, age[i] + grid[i], date[i] + grid[i])
          \partial \Lambda_n = \lambda_n * \partial t[i]\Lambda_{\rm b} += \partial \Lambda_{\rm n}W_p = exp(\Lambda_p)\partial N_{p}[i] += \partial N_{p} * w_{p}Y_e[j] += W_pend
     \partial N_e[T_i] += W_p * \Delta[i]\partial V[T_i] += W_p^2 * \Delta[i]end
Y_p .= Y_eend
```
Figure 1: The Pohar Perme function in the NetSurvival. il package looks like the formula itself.

Note: The daily_hazard function is thoroughly optimized. 60% of runtime in exp ! JuliaCon 2024 / R. Alhajal, O. Laverny / [Standard estimators in the field](#page-6-0) $6/27$

The **Grafféo log-rank test³ is designed to compare net survival functions across multiple groups**. There is a stratified version of this test as well.

Definition (Grafféo log-rank test)

The null H_0 hypothesis tests the following assumption:

$$
\forall t \in [0, T], \ \Lambda_{E,g_1}(t) = \Lambda_{E,g_2}(t) = ... = \Lambda_{E,g_k}(t)
$$

where $G = \{g_1, ..., g_k\}$ is a partition of 1, ..., n consisting of disjoint groups of individuals that we wish to compare to each other.

³Nathalie Grafféo, Fabienne Castell, Aurélien Belot, and Roch Giorgi. "A Log-Rank-Type Test to Compare Net Survival Distributions". In: Biometrics 72.3 (Jan. 2016), pp. 760–769. ISSN: 0006-341X. DOI: [10.1111/biom.12477](https://doi.org/10.1111/biom.12477).

Definition (Grafféo log-rank test)

For all groups $g \in G$, let's denote the numerator and denominator of the Pohar Perme (partial) excess hazard estimators, restricted to individuals in the group, by:

$$
\partial N_{E,g}(s) = \sum_{i \in g} \frac{\partial N_i(s)}{S_{P_i}(s)} - \frac{Y_i(s)}{S_{P_i}(s)} \partial \Lambda_{P_i}(s)
$$

$$
Y_{E,g}(s) = \sum_{i \in g} \frac{Y_i(s)}{S_{P_i}(s)}
$$

$$
R_g(s) = \frac{Y_{E,g}(s)}{\sum_{g \in G} Y_{E,g}(s)}
$$

Then, define the vector $\mathsf{Z} = (Z_{\mathsf{g}_r} : \ r \in 1,...,k-1)$ with entries:

$$
Z_{g}(T) = N_{E,g}(s) - \int_0^T Y_{E,g}(s) \partial \hat{\Lambda}_E(s)
$$

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Definition (Grafféo log-rank test)

The test statistic is then given by:

$$
\mathit{U}(\mathcal{T})=Z(\mathcal{T})'\hat{\Sigma}_Z^{-1}Z(\mathcal{T})
$$

where the entries of the $\hat{\Sigma}_Z$ matrix are given by:

$$
\hat{\sigma}_{g,h}(T) = \int_0^T \sum_{\ell \in G} (\delta_{g,\ell} - R_g(t)) (\delta_{h,\ell} - R_h(t)) \left(\sum_{i \in \ell} \frac{\partial N_i(s)}{S_{P_i}^2} \right)
$$

Under H_0 , the statistic $U(\mathcal{T})$ is asymptotically $\chi^2(k-1)$ -distributed. We reject the H_0 hypothesis when the p-value obtained is under a certain value depending on the error rate chosen, thus admitting the notable difference between the groups.

Definition

The crude mortality rate is the global mortality rate for a population. It can be separated by cause of death, such as:

$$
M_E(t) = \int_0^t S_O(u-) \partial \Lambda_E(u)
$$

Note: We denote $M_P(t) = \int_0^t S_O(u-) \partial \Lambda_P(u) du$ and $F_O(t) = M_E(t) + M_P(t)$.

Lemma (The Cronin-Feuer estimator 4)

To estimate the quantity above, we first introduce the Cronin-Feuer estimator given by:

$$
\hat{M}_E(t) = \int_0^t \hat{S}_O(u-) \partial \hat{\Lambda}_E(u)
$$

⁴Kathleen A Cronin and Eric J Feuer. "Cumulative cause-specific mortality for cancer patients in the presence of other causes: a crude analogue of relative survival". In: Statistics in medicine 19.13 (2000), pp. 1729–1740.

The nessie function allows us to calculate the estimated sample size by yearly intervals as well as the average lifespan left.

Definition (Estimated Sample Size or ESS)

The estimated sample size is given by: $\quad ESS(t)=\sum_i^N S_{P_i}(t)$

Definition (expected lifespan)

Due to the constance of the hazard rates on each cell of the lifetable, the life expectation can be computed through the following formula⁵:

$$
E(P) = \int_0^\infty S_P(t) \partial t = \sum_{j=0}^\infty \frac{S_P(t_j)}{\lambda_P(t_j)} \left(1 - e^{-\lambda_P(t_j)(t_{j+1} - t_j)}\right)^{-1}
$$

Note: This feature is implemented in the RateTables.jl package and depends on the Distributions.expectation function.

⁵Per Kragh Andersen. "Life years lost among patients with a given disease". In: Statistics in medicine 36.22 (2017), pp. 3573–3582.

[Comparison with the](#page-14-0) R/C++ [standard](#page-14-0)

Lines of code: R VS Julia

With Julia, the code is concise and easy to read, half of it is docs:

 \vee NFTSURVIVAL $src >$ & CrudeMortality.il $> ...$ \sim \sim \sim \angle .aithub $\overline{1}$ $\overline{2}$ CrudeMortality \sum vecode $\overline{\mathbf{z}}$ > henchmark This method calculates the crude mortality and presents both the excess morta \overline{A} \geq data $\overline{\mathbf{5}}$ \vee does 6 The default Cronin-Feuer estimator can be fitted to data with the following i \sum build $\overline{7}$ \mathbf{g} fit(CrudeMortality, args...) \vee cre \mathbf{Q} \vee accatc where the 'args' are passed to 'fit(EdererII, args...)' to compute the excess $10₀$ # citations.css 11 \equiv references bib 12 A more direct syntax can be used, specifying directly the estimator for the e Liberiches md 13 \blacktriangleright example.md 14 CrudeMortality(fit(EdererII.args...)) ♦ getting started.md 15 To use another excess hazard estimator, simply replace 'EdererII' with the me 16 \blacktriangleright index.md 17 # references.md References: 18 & make.il 19 * [cronin2000cumulative](@cite) Cronin, Kathleen A and Feuer, Eric J (2000). **D** Manifest toml 0.000 20 **D** Project.toml 21 struct CrudeMortality \vee src M.::Vector{Float64} 22 & CrudeMortality.jl 23 M.::Vector{Float64} M_p::Vector{Float64} 24 & EdererI.il 25 function CrudeMortality(npe::NPNSEstimator{Method}) where Method & EdererII.jl 26 $S_0 = cumprod(1 - npe.\partial\Lambda_0)$ & fetch_datasets.jl 27 $M_e = \text{cumsum}(npe.\partial\Lambda_e : S_o)$ & GraffeoTest.il 28 $M_p = \text{cumsum}(npe.\partial A_p : S_o)$ & Hakulinen.jl 29 return new(M. .+ M., M., M.) JuliaCon 2024 / R. Alhajal, O. Laverny en Comparison with the R/C++dstandard 12/27

NetSurvival.jl files

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With Julia, the code is concise and easy to read, half of it is docs:

\vee NETSURVIVAL src > \triangle Nessieji > Ξ Nessie > Ω Nessie struct Nessie $\frac{1}{2}$.aithub expected sample size::Vector{Eloat64} $\overline{2}$ \sum vscode \overline{a} expected life time::Float64 > henchmark \overline{A} grid::Vector{Float64} \geq data k function Nessie(T. A. age, year, rate preds, ratetable) \vee docs 6 annual grid = 0:RateTables.RT DAYS IN YEAR:maximum(T) \sum build \overline{z} exp spl size = zeros(length(annual grid)) \mathbf{a} exp life time = 0.0 \sim ere α for i in eachindex(age) \vee accets $10₀$ $P_i = \text{Life(ratetable(rate preds[i,:]...]}$, age[i], year[i]) # citations.css 11 for i in eachindex(annual grid) F references.bib 12 exp spl size[i] += $ccdf(P_1, annual grid[i])$ # benches.md 13 end \blacktriangleright example.md 14 exp life time += expectation(P_1) # getting started.md 15 end 16 return new(exp spl size, exp life time / RateTables, RT DAYS IN YEAR / length(age), annual grid) M. index md 17 end # references.md 18 end & make.il 19 **O** Manifest.toml 1.11 20 **C** Project.toml 21 nessie \vee src. 22 & CrudeMortality.jl $2³$ To call this function, use the formula below: 24 & Edererl.il 25 nessie(@formula(Surv(time,status)~covariate), data, ratetable) & EdererII.il 1.11 26 & fetch_datasets.jl 27 function $nessie(args...)$ & GraffeoTest.il 28 $r = fit(Nessie, args...)$ & Hakulinen.jl 29 if (typeof(r)<:Nessie) & Nessie.il 38 neturn n 31 end & NetSurvival.jl 32 transform!(r. :estimator => ByRow(x-> (x.grid. x.expected life time. x.expected sample size)) => [:gri & NPNSEstimator.il 33 select!(r, Not(:estimator)) **& PoharPerme.il** 34 & Surv_and_Strata.jl 35 $lt = deepcopy(r)$ \times test 36 select!(lt, Not([:expected sample size, :grid])) & runtests.jl 37 JuliaCon 2024 / R. Alhajal, O. Laverny / [Comparison with the R/C++ standard](#page-14-0) 11fe_time)
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NetSurvival.jl files

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On the other hand, most of the R and C++ files from R:: relsurv span over thousands of lines of code:

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// if this person died at this time

The cloc software yields the following numbers for each repository:

Table 1: Summary of the count of lines of code for different languages in relsurv

Table 2: Summary of the count of lines of code in NetSurvival.jl

A glimpse in our benchmarks for a few standard algorithms:

Table 3: Runtime multipliers comparing NetSurvival. jl to R:: relsurv, computed on a i9-13900 processor. The data used is the colrec dataset and the slopop mortality table.

Example: The Pohar Perme function on Julia takes ≈ 0.11 seconds to run whereas R takes ≈ 2.27 . Remark: One key advantage the NetSurvival. jl package has on R:: relsurv is the function that fetches the daily hazard rates from life tables and matches them to the individuals from the dataset, hosted in the JuliaSurv/RateTables.jl. Our implementation is blazing fast w.r.t. the original $C++$ one.

[Real data showcase](#page-25-0)

Cohort: colrec

- (i) 5971 patients
- (ii) Colon or rectal cancer diagnosis between 1994 and 2000
- (iii) 7 variables: age, year, sex, status, follow-up time, cancer stage, and cancer site.
- (iv) Sourced from the Slovenian cancer registry

Rate table: slopop

- (i) Slovenian mortality table
- (ii) Includes information on age, year, and sex.
- (iii) Extracted from official census mortality rates.

Let's take a closer look at those variables:

Figure 4: Histograms showing the distributions of age and the follow-up time (in years).

Given the cohort colrec and the related mortality table slopop, we will apply the Pohar Perme estimator using:

Julia code using NetSurvival, DataFrames, RateTables pp_estimator = fit(PoharPerme, $@formula(Surv(time,status)^{-1})$,colrec,slopop)

By compiling the code above, we get a table with the net survival probability in daily intervals. Let's plot the results for better visualization.

Output of the Pohar Perme estimator

Figure 5: Pohar Perme net survival estimator. Right: only the first 5 years.

The graph above shows that, with time, the estimator loses a lot of its accuracy. Let's find out why.

Looking at the previous histograms, we can see that the age variable is important. The nessie function gives an estimated of expected sample size w.r.t. cancer only:

Julia code

elt, ess = nessie(@formula(Surv(time,status)~age65), colrec, slopop)

Output:

Table 4: Estimated sample size by year for patients above and under 65 years old.

nessie function output: ess object

Again, for a better understanding of these values, we present them in the graph below:

Figure 6: Graph representation of the estimated sample size for younger and older patients in yearly intervals.

Conclusion: We choose to censor the data after 5 years.

As for the expected years left for these two groups, we get the output below:

Table 5: Expected life time for patients grouped by age.

Around 5 years into the study, the crude mortality is calculated:

Output at time $= 5$ years:

Table 6: Crude mortality at year 5.

This shows that of the 64% patients that have died, 53% is due to colorectal cancer while 11% is due to other causes.

Julia code

When applying the Grafféo test, we find that only two variables carry real significance in the study.

test age = fit(GraffeoTest, @formula(Surv(time5,status5)~age65), colrec, slopop) test_stage = fit(GraffeoTest, @formula(Surv(time5,status5)~stage), colrec, slopop)

They are unsurprisingly age65 and stage, with p-values of $1.85 * 10^{-18}$ and $7.18 * 10^{-237}$ respectively for the first 5 years of the study.

Conclusion: In the colrec dataset, and with reference to the slopop mortality table, age and cancer stages are, unsurprisingly, the more important variables in a net survival context. The older and the more advanced the cancer is, the lower the risk of survival.

[Conclusion](#page-36-0)

The JuliaSurv organisation on GitHub was created to house all the packages related to survival analysis in one place in an attempt to grow the Julia general registry within that context, and to keep track of older packages.

So far, it comprises:

```
(i) NetSurvival.jl
```

```
(ii) RateTables.jl
```

```
(iii) SurvivalBase.jl
```

```
(iv) SurvivalDistributions.jl.
```
More to come in the near future!

To summarize:

- (i) Survival analysis deals with (censored) data where the variable of interest is the time until a certain event occurs (e.g., death).
- (ii) In some specific cases, in particular for cancer registries, the exact cause of death of each individual is unavailable and/or unreliable.
- (iii) Relative Survival is a theory built to handle this issue.
- (iv) Our Julia implementation is easier to read, more concise, and faster than R's, thus, making it future-proof.
- (v) The JuliaSurv organisation has a bright future ahead!

Contributions to NetSurvival.jl as well as to JuliaSurv are more than welcome!

JuliaSurv/NetSurvival.jl

Star it on Github :)